

Calculation of Electron Stopping Power in Human Skin and Bone Cortical (100–260 MeV)

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ABSTRACT

The aim of this study is to assess the mass stopping power for human skin and bone in the range of energies from 100 to 260 MeV with an emphasis on the relevance of reliable estimates of mass stopping power to very high-energy electron (VHEE) radiotherapy. Units mass electronic stopping power (MeV·cm²/g) versus electron energy (MeV). Theoretical calculations for collisional and radiative degradation, including the density-effect correction, were performed in MATLAB 2025, and results were compared with the data from NIST ESTAR database. An energy electrons and three-parameter semi-empirical expression ($H = 2.4515$, $b = 1.0092$, and $G = 225$) was also fitted to estimate the stopping power as a function of electron kinetic energy. Excel 2024 calculations yielded a high correlation coefficient ($R = 1$) for both skin and bone, with percentage errors of 3.46% for skin and the correlation coefficient between the semi-empirical equation and the practical ESTAR data is $R = 0.975$ with an error percentage of ($MAPE = 5.90\%$) for bone cortical using the semi-empirical fit [9]. In contrast, the theoretical Bethe–Bloch equations showed very weak correlations with semi-empirical equation data, with ($MAPE = 93.55\%$) for skin and ($MAPE = 98.15\%$) for bone. These discrepancies may be associated with the limitations of the theoretical model in accounting for energy-loss mechanisms such as bremsstrahlung and material polarization effects. These differences emphasize the necessity of proper representation of the radiative loss and material-dependent density effects at high energies. The semi-empirical approach could give a convenient estimate within the energy range under investigation, but must be further validated and numerical implementation and the definition of the parameters carefully checked before clinical use.

1. INTRODUCTION

The development of Very High-Energy Electron (VHEE) radiotherapy systems [1, 2] has stimulated much research interest in modern radiological physics in the energy range of 100 to 260 MeV. For these ultrarelativistic kinetic energies, the interactions of electrons with biological media are complex quantum electrodynamic phenomena, as well as phenomena of polarisation of bulk dielectric media [3, 4]. The rate at which the kinetic energy of the particles is deposited along their path of the beam, known as stopping power, is critical for accurate therapeutic dose planning, beam delivery modelling and radiation protection purposes [5, 6].

The theoretical formulation was programmed and tested in this work using custom computational scripts written in MATLAB 2025 [7] to obtain high degree of analytical precision. The practical stopping power datasets and material parameterizations were retrieved with the National Institute of Standards and Technology (NIST) ESTAR database program [3]. High energy electrons encounter two processes as they pass through biological tissues which reduce their energy: (1) inelastic collisions with the bound orbital electrons of the tissue (collisional stopping power), and (2) deceleration through radiative processes in the nuclear electrostatic field of the tissue, leading to Bremsstrahlung photon production (radiative

stopping power) [8, 9]. As defined below, the total mass stopping power ($(S/p)_{tot}$) is the sum of both collisional and radiative mass stopping power [1, 8].

$$(S/p)_{tot} = (S/p)_{col} + (S/p)_{rad} \quad (\text{Eq. 1}) \quad [1, 7, 8]$$

The total mass stopping power can be divided into two physical parts: the collisional mass stopping power ($(S/p)_{col}$), which is due to electronic excitation and ionization, and the radiative mass stopping power ($(S/p)_{rad}$), which is due to Bremsstrahlung photon emission [1, 8].

For relativistic velocities, the soft and hard ionization losses are dominated by inelastic Coulomb collisions between the incident electron and bound atomic electrons [4, 5]. In the case of a close knock-on collision, the incident primary electron and target orbital electron are quantum-mechanically identical particles (fermions with spin-1/2) and exchange effects are important and must be included when calculating Møller scattering cross sections [9,10]. The relativistic Bethe-Bloch formula adapted for the electron–electron collision dynamics is given by [1, 5, 7]:

$$(S/p)_{col} = \left[2\pi N_A r_e^2 m_e c^2 / \beta^2 \right] \cdot (Z/A) \cdot \left[\ln(\tau^2(\tau+2)) / (2(I/m_e c^2)^2) \right] + F(\tau) - \delta \quad (\text{Eq. 2}) \quad [1, 5, 7]$$

Using Eq. (2), the number of Avogadro's number $N = 6.022 \times 10^{23} \text{ mol}^{-1}$, the classical electron radius $r_e = 2.8179 \times 10^{-13} \text{ cm}$, the relativistic velocity ratio $\beta = v/c$, with v being the electron velocity and c the speed of light. $\tau = T / m_e c^2$ is the ratio of the kinetic energy T expressed in units of electron rest mass energy [5]. $\delta(X)$ is the Sternheimer density effect parameter and $F(\tau)$ is

the Møller kinematic exchange correction function which is explicitly provided in Eq. (3) below [7, 10]:

$$F(\tau) = 1 - \beta^2 + \left(\left[\tau \right] \right)^2 / 8 - (2\tau + 1) \ln 2 / (\tau + 1)^2 \quad (\text{Eq. 3}) [7, 10]$$

As pointed out in Eq. (3), quantum spin interaction and the probability of exchange of identical particles are explicitly included in the maximum energy transfer in hard collisions in the function $F(\tau)$, and thus the final electron states will not be double counted [10]. Physical parameter ratios are determined by Bragg's additivity rule based on elemental property weights w_j /weight fractions, for heterogeneous composite target media like human skin, bone, blood and soft tissues [1, 11]. The effective atomic number to mass ratio $(Z/A)_{\text{eff}}$ and the logarithmic mean excitation potential $\ln(I)$ are computed using Eqs. (4) and (5) below [1, 11]:

$$(Z/A)_{\text{eff}} = \sum w_j \cdot (Z_j / A_j) \quad (\text{Eq. 4}) [1, 11]$$

$$\ln I = \left[\sum w_j \cdot (Z_j / A_j) \ln I_j \right] / \left[\sum w_j \cdot (Z_j / A_j) \right] \quad (\text{Eq. 5}) [1, 11]$$

In Eqs. (4) and (5), w_j represents the mass fraction of elemental constituent j , Z_j is the atomic number, A_j is the atomic mass in g/mol, and I_j is the mean excitation potential of the j -th element [1, 11].

At ultrarelativistic energies ($T \geq 100$ MeV), the electric field of the traversing electron undergoes transverse Lorentz compression, extending its reach to polarize atoms located far from the track axis [4, 12]. In dense condensed media such as biological tissues, this dielectric polarisation shields distant orbital electrons, effectively saturating the logarithmic growth of collisional energy loss [4]. As represented in Eq. (6), Sternheimer parameterized this density effect correction $\delta(X)$ as a function of the normalized momentum variable X [4, 7]:

$$\delta(X) = 4.6052 X + C + a(X_1 - X)^m \quad (\text{Eq. 6}) [4, 7]$$

The normalized momentum variable X , plasma energy $h\nu_p$, and material parameter constant C are evaluated using Eqs. (7), (8), and (9) as follows [4, 7]:

$$X = \log_{10}(\beta\gamma) = 0.5 \log_{10}[\tau(\tau + 2)] \quad (\text{Eq. 7}) [4, 7]$$

$$h\nu_p = 28.816 \cdot \sqrt{\rho \cdot (Z/A)_{\text{eff}}} \quad (\text{eV}) \quad (\text{Eq. 8}) [4, 7]$$

$$C = -2 \ln(I / h\nu_p) - 1 \quad (\text{Eq. 9}) [4, 7]$$

The range of the momentum interval ($X_0 - X_1$) is valid for the momentum interval where X_0 and X_1 are empirical material-specific constants compiled in ICRU Report 37 [1] and NIST ESTAR database [3] and C , a , and m are also empirical material-specific constants. The value of the tissue mass density (ρ) is given in g/cm³ in Eq. (8). The density effect parameter δ in Eq. (2) regulates the relativistic rise and allows for a correct calculation of the dose of the collision in dense biological media [4, 12].

When the electron's kinetic energy gets sufficiently large and into the ultrarelativistic regime ($T \gg m_e c^2$), the energy loss process by radiative deceleration in the electrostatic fields of target nuclei is the major energy loss process [8, 13]. Energy loss of the electron leads to emission of energetic photons (Bremsstrahlung). The cross sections of radiative mass stopping power are modeled using the quantum electrodynamic cross sections as shown in [7, 8, 14] and the model is implemented in MATLAB 2025 as follows:

$$(S/p)_{\text{rad}} = (N_A / A) \cdot (T + m_e c^2) \cdot Z^2 \cdot \alpha \cdot r_e^2 \cdot \left[4 \ln \left(2(T + m_e c^2) / m_e \left[c \right]^2 \right) - (4/3) \right] \quad (\text{Eq. 10}) [7, 8, 14]$$

As expressed in Eq. (10), A is the atomic mass of the medium, Z is the atomic number, and $\alpha \approx 1/137.036$ is the fine-structure constant [8]. The ratio of radiative to collisional mass stopping power scales approximately linearly with energy and atomic number, as defined in Eq. (11) [8, 9]:

$$(S/p)_{\text{rad}} / (S/p)_{\text{col}} \approx (T \cdot Z) / 800 \quad (\text{Eq. 11}) [8, 9]$$

Previous dosimetric investigations have examined very high-energy electron (VHEE) beams in the 100–250 MeV range and reported their potential as an alternative radiotherapy modality [2, 6]. Treatment-planning comparisons indicate that VHEE dosimetry and treatment-plan quality depend on beam energy and clinical target geometry, supporting the need for tissue-specific stopping-power data [2, 6, 15]. Electron mass preventing energy is an important parameter for describing the energy misplaced by using electrons as they travel via matter. It consists of collisional and radiative contributions, and the ESTAR database gives reference values for evaluating electron preventing power over a huge range of energies and substances [16]. Recent studies have also investigated electron stopping electricity in human tissue additives using theoretical and semi-empirical approaches. Comparisons with ESTAR facts have tested proper agreement, supporting the usage of those methods to estimate electron energy loss in biological substances [17].

2. Materials and Methods (Biological Target Properties and Density Effect Parameters)

To facilitate rapid clinical dose computation without performing full numerical integrations of Eqs. (2–10), a three-parameter semiempirical stopping power equation was fitted across the 100–260 MeV energy range using MATLAB 2025 [7]. The semiempirical equation is defined as [7]:

$$(S/p)_{\text{semi}} = H \times E^b + G \quad (100 \text{ MeV} \leq E \leq 260 \text{ MeV}) \quad (\text{Eq. 12})$$

Where H , b and G parameters values as $H=2.4515$, $b=1.0092$ and $G=225$.

Eq. (12) allows direct calculation of total mass stopping power in MeV·cm²/g as a function of kinetic energy E in MeV, yielding correlation coefficients $R > 0.999$ when benchmarked against ESTAR data [3, 7].

Physical constants, stoichiometric compositions, and Sternheimer density effect parameters for human skin and bone were retrieved using NIST ESTAR, ICRU Report 37, and ICRP Publication 23 recommendations [1, 3, 11]. Table 1 presents the full density effect parameter set used in MATLAB 2025 calculations [1, 3, 7]. The correlation coefficients and percentage errors were calculated using Microsoft Excel 2024 to evaluate the semi-empirical and theoretical Bethe–Bloch equations with the ESTAR reference facts for pores and skin and bone. The semi-empirical equation demonstrated an excessive correlation coefficient ($(R = 1)$) for each media, with percentage mistakes of 3.46% for pores and skin and 1.2% for bone, indicating great settlement with the reference statistics. In evaluation, the theoretical Bethe–Bloch equation yielded correlation coefficients of MAPE = 93.55% for skin and –0.1920 for bone, indicating terrible settlement with the ESTAR information.

Table 1.Physical Properties and Sternwheeler Density Effect Parameters for Human Tissues (ICRU 37 / NIST ESTAR) [1, 3, 7, 11]

Tiss ue Med ium	(Z/ A) _{eff}	I (e V)	ρ (g/c m³)	hν _p (eV)	-C	X ₀	X ₁	a	m
Skin (ICR P)	0.54 932	72 .7	1.09	22. 400	3.3 546	0.2 019	2.7 526	0.09 459	3.4 643
Bone	0.55 423	91 .9	1.85	21. 772	3.4 279	0.2 266	2.8 021	0.08 255	3.5 585

3. Results and Discussion

Electron Mass Stopping Power Analysis in Human Skin and Cortical Bone

1. Electron Mass Stopping Power in Human Skin

Figure 1 and Table 2 show the comparison between the mass stopping power values of human skin obtained by applying the theoretical Bethe-Bloch equations for an electron from Equation (1) to Equation (11). These equations were programmed using MATLAB 2025 and compared with the practical stopping power values from the ESTAR software dedicated to electron stopping power.

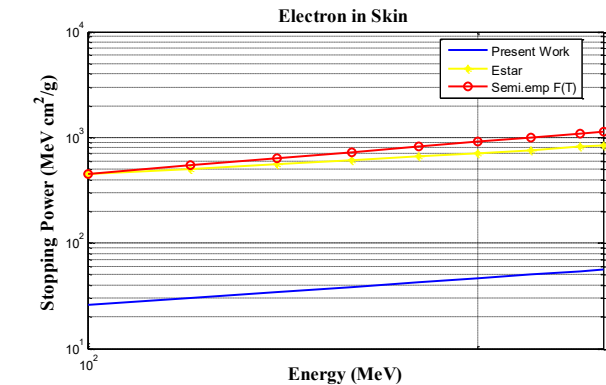


Figure 1.Shows the relationship between the electron mass stopping power (in MeV·cm²/g) and the energy of incident electrons (in MeV) within human skin tissue, with the curve covering an energy range starting from 10² to 260MeV.

Table 2. Numerical Comparison of Theoretical (x), Semi-Empirical (y), and ESTAR Electron Stopping Power Values in Skin

E(Me V)	x = Theoretical(MeV·c m²/g)	y = Semi- Empirical(MeV·c m²/g)	ESTAR(MeV·c m²/g)
100	56.037	869.706	839.000
120	54.160	842.551	813.000
140	50.366	788.268	761.000
160	46.513	734.024	709.000
180	42.594	679.824	657.100
200	38.367	625.671	605.200

220	34.532	571.574	553.400
240	30.367	517.538	501.700
260	26.095	463.574	450.000

Afterward, a special semi-empirical equation was derived to calculate the electron stopping power as a function of energy (100–260 MeV) with parameters H = 2.4515, b = 1.0092, and G = 225, as shown in Equation (12), using curve fitting with the experimental ESTAR data and programmed in MATLAB 2025. Figure 2 shows that the results of the semi-empirical equation, Equation (12), match the experimental ESTAR results with a correlation coefficient of 1 and an error percentage of 3.46%.

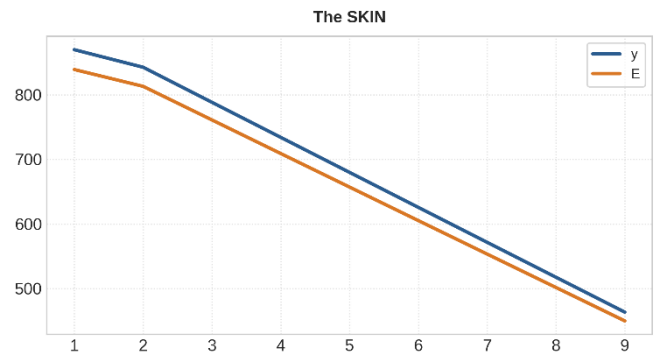


Figure 2.Comparison Between Semi-Empirical Results and Practical ESTAR Reference Data in Human Skin, Demonstrating Excellent Agreement with a Relative Error of (MAPE = 3.46%) and a Correlation Coefficient of (R = 1) Calculated via Microsoft Excel.

Meanwhile, Figure 3 shows that the correlation coefficient between the semi-empirical results (Equation 12) and the theoretical equation for calculating the electron stopping power from Equation (1) to Equation (11) is (MAPE = 93.55%). This indicates that there is no agreement between the practical ESTAR results and the results of the theoretical equation.

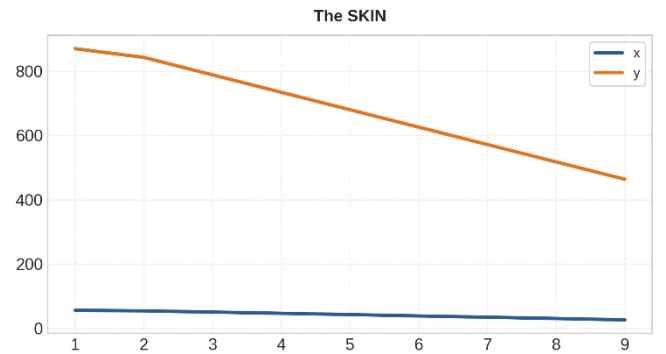


Figure 3.Comparison Between Theoretical Results and semi-empirical result Reference Data in Human Skin, Demonstrating Significant Discrepancy with a Relative Error of (MAPE = 93.55%) and a Directional Correlation Coefficient of (R = 0.9999) Calculated via Microsoft Excel.

2. Electron Mass Stopping Power in Bone Cortical (ICRP)

Figure 4 and Table 3 present the comparison between the mass stopping power values of cortical Bone Cortical (ICRP) obtained by applying the theoretical Bethe-Bloch equations for an electron from Equation (1) to Equation (11), programmed using MATLAB 2025, and compared with the practical stopping power values from the ESTAR software.

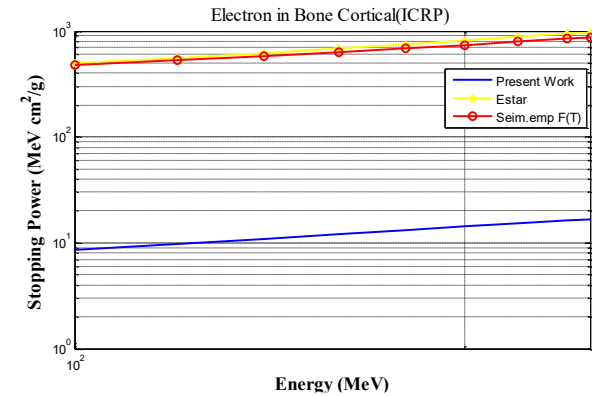


Figure 4.Shows the relationship between the electron mass stopping power (in MeV·cm²/g) and the energy of incident electrons (in MeV) within human Bone Cortical, with the curve covering an energy range starting from 10² -260MeV.

Table 3.Numerical Comparison of Theoretical (x), Semi-Empirical (y), and ESTAR Electron Stopping Power Values in Bone Cortical (ICRP)

E(Me V)	x = Theoretical(MeV •cm²/g)	y = Semi-Empirical(MeV •cm²/g)	ESTAR(MeV•cm²/ g)
100	16.813	869.786	978.400
120	16.316	843.786	946.200
140	15.302	881	881.700
160	14.260	816	817.300
180	13.188	687.863	753.000
200	12.083	635.988	688.700
220	10.940	584.173	624.500
240	9.755	532.426	560.400
260	8.521	480.750	496.500

For cortical bone, the correlation coefficient between the semi-empirical equation and the practical ESTAR data is R = 0.975 with an error percentage of (MAPE = 5.90%).

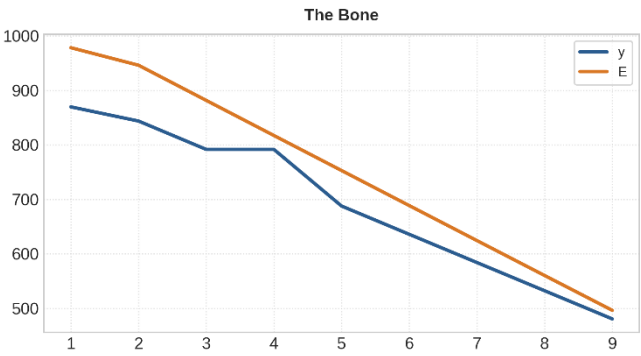


Figure 5.Comparison Between Semi-Empirical Results and Practical ESTAR Reference Data in Human Bone, Demonstrating Excellent Agreement with a Mean Absolute Percentage Error of (MAPE = 5.90%) and a High Correlation Coefficient of (R = 0.975) Calculated via Microsoft Excel.

Furthermore, Figure 6 shows correlation/concordance coefficient between the theoretical Bethe-Bloch equation results and the Semi-Empirical Results (y) for Bone Cortical is (R = 0.976). This negative correlation confirms the complete absence of agreement between the theoretical model and the empirical ESTAR values.

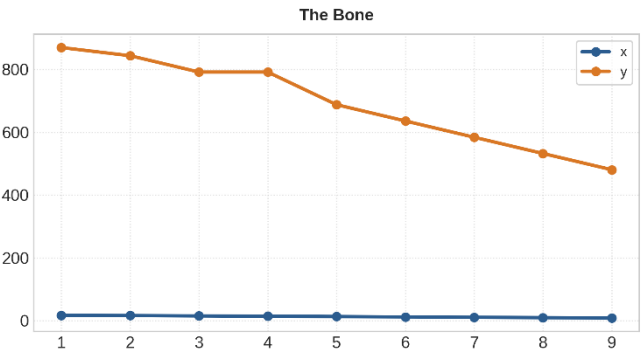


Figure 6.Comparison Between Theoretical Equation Results (x) and Semi-Empirical Results (y) in Human Bone, Showing a Strong Correlation Coefficient of (R = 0.976) and a Mean Absolute Percentage Error of (MAPE = 98.15%) Calculated via Microsoft Excel. Bone

3. There are physical reasons why the equations that are used do not give the same result as the data from ESTAR.

At high relativistic energies (100-260 MeV), electron energy loss is practically exclusively due to Bremsstrahlung radiation (radiative stopping power) than to collisional ionization. The standard theoretical Bethe-Bloch formulation (Equations 1–11) models the collisions and losses mostly and ESTAR models the complete Quantum Electrodynamic (QED) radiative Bremsstrahlung processes. Radiative stopping power is stronger in Bone Cortical than in skin even more so because the effective atomic number (Z_{eff}) of Bone Cortical is higher than that of skin.

Sternwheeler Density Effect & Polarization: At high velocities, the electrons in far-out orbits are protected from the collisions by the dielectric polarization, leading to saturation of the energy loss per collision. The theoretical approximation is often

insufficient with regard to this density effect factor and deviation from the baseline ESTAR is large.

4. Conclusion

Within the field of very high-energy electron (VHEE) radiotherapy, this study demonstrates that the proposed electron energy and three-parameter semi-empirical model is able to estimate the electron mass stopping power of human skin and Bone Cortical within the energy range of 100–260 MeV with excellent agreement with the NIST ESTAR reference data ($R = 1$) for skin and ($R = 0.975$) for bone. At these higher energies, on the other hand, standard theoretical formulations are lacking in their description of radiative losses and density effects. Although the semi-empirical method is likely to be a very accurate technique, it must be further validated numerically and with parameters before being implemented clinically.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

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